Mental arithmetic skills for the early grades

These mental math skills greatly ease the rest of a child's progress in mathematics. See articles on http://thinkmath.edc.org "Addition and subtraction" and "Language and mathematics" for more about the ideas behind these skills. Under "mental math," see article on "Fact of the day" for a technique that helps children master the skills through focused "drill and thrill" practice rather than random "drill and kill."

Kindergarten

1. **Hop on number line**, noticing the start, jump size and direction, and end.
2. **Remember** where they started, how far they went.
3. **Copy finger numbers** from visual memory.
4. **Say how many fingers are "down"** as well as "up"

First grade Every first grader can and should learn these six skills with total fluency.

1. **Count backwards** from 40 to 0;
2. **Double numbers** up through 12;
3. **Find half of even numbers** that are 20 or less;
4. **Make "pairs to 10"** (e.g., if you say 6, they respond with 4);
5. **Add 10** (find 10 more) to any number from 0 through 90, and **subtract 10** from any number from 10 through 100; and
6. **Use knowledge** of addition for related subtraction (e.g., if I start with 5 and end with 8, what must I have added?). Continue to see everything on number line, not using the line for *answers* but to make sense of the operations. Use *language* knowledge to hear "two hundred plus three hundred" spoken (not written) and respond "five hundred."

All other facts build off of these ***mental*** *arithmetic skills*. If children master only these in grade 1, they are well prepared for grade 2. The essential *arithmetic* skills in first grade are all *mental*; no formal algorithms yet.

Second grade Virtually *all* children can master *all* these skills by the end of grade 2.

Revisit first-grade skills only as needed and then build on them. The intuitions developed with these mental calculations lead directly to the arithmetic properties and algorithms.

1. **Addition/subtraction and place value:** Over the course of the year, pairs to 10 (e.g., 3, 7) lead to pairs to 100 (e.g., 30, 70) and 1000 (e.g., 300, 700). Children grow comfortable with pairs that use ½, e.g. 4 → 6, and then 4½ → 5½
2. **Addition/subtraction:** Children who are fluent with pairs to 10 quickly become fluent with pairs to 20 early in the year, and extend the idea to pairs to 30.
3. **Addition/subtraction:** By the end of second grade, they can add or subtract 8 or 9 (or 11 or 12) by being facile at adding and subtracting 10 and adjusting. (E.g., to subtract 8 from 35, they think "35 – 10 = 25, but then I've subtracted too much, so," because they know that 8 and 2 are 10, "I must put back 2, and the answer is 27.")
4. **Addition/subtraction:** Invent pairs to 11, or to 9, by using pairs to 10 and adjusting.
5. **Addition/subtraction:** Deconstruct numbers under 10 as "5 + more," and regroup to "explain" sums like 8 + 7. E.g., 8 + 7 = (5 + 3) + (5 + 2) = (5 + 5) + (3 + 2). (See http://thinkmath.edc.org/index.php/Addition_and_subtraction#Four-hand_addition .)
6. **Subtraction and place value:** By mid-year, they can solve 23 – 20 by applying a *linguistic* idea to a mathematical context. If "Laura Lee minus Laura = Lee" and "Laura Lee minus Lee = Laura" then if one's name is "twenty three," "twenty three – twenty = three" and "twenty three – three = twenty." (See explanation at "language and mathematics" on the website.)
7. **Locate 71½** (and other such numbers, using ½ as the only fraction) on a number line.
8. **Multiplication/division:** By the end of the year, they double numbers mentally through 100 and halve "easy" even numbers (all digits even) through 800. (Half of 860 is half of 800 and half of 60.)
9. **Tiny multiplication with arrays and intersections:** Using intersections (see explanation at http://thinkmath.edc.org/index.php/Multiplication#Intersections_to_clarify_multiplication_by_0_and_1) students understand *why* 0× anything is 0 and that 1× any number is that number. They know that "2×

a number" is the doubling that they already do well; they know that a number × 10 is that many tens (dimes), and they know what value that is; and they recognize, by sight, these arrays 3×3, 3×4, 3×5, 3×6, 4×4, 4×5, 4×6, 5×5, and consequently (implicitly) know those number facts.

Third grade

Build on 2nd grade mental computation: pairs to 10; adding and subtracting 10s; equal comfort with 70 + 80 as with 7 + 8; ability to do 1000 – 7 *mentally*, and to solve 1000 – 27 by thinking 1000 – 7 and then subtracting 20 from that (or subtracting 20 and then 7); deconstructing/regrouping numbers to be able to add and subtract two digit numbers with ease, *mentally*; total comfort with all second grade multiplication facts and ideas (understanding 0 × anything, 1 × anything, 2 × a number is its double…). Extend these *mental* computation abilities to the following:

1. **Double two-digit numbers** and take ½ of any (even) two-digit number;
2. **Visualize number line** to picture 76 – 48 as distance from 48 to 76, via 50 and 70;
3. **Multiply any number by 4 or 8 by** doubling the appropriate number of times (to multiply 4, double twice; to multiply by 8, double three times);
4. **Multiply any number by 5 by** *understanding* that the result is ½ of multiplying that number by 10; use this skill both to help with 5 × facts and to multiply 5 × easy (both-digits-even) two-digit numbers *mentally* (e.g., 5 × 84 is half of 10 × 84);
5. **Know square numbers (3×3, 4×4, 5×5, 6×6, ... 12×12)** like their best friends' names;
6. **Learn the remaining multiplication facts fluently** (only six facts left to memorize: 6×7, 6×8, 6×9, 7×8, 7×9, 8×9).
7. **Picture fractions *on the number line***, including "mixed numbers"

Very fast one-a-day method for multiplication facts

A child is asked to be the "class specialist" for *one* math fact for that day. Any time anyone needs help with that fact -- or any time anyone (including the teacher) asks for that fact -- that child is the expert-of-the-day in charge.

Method

At the beginning of the day, "publicly" assign *one* math fact to each of three children who become that day's "experts," each "specializing" on just one fact. For example, one might be in charge of 7 × 8 = 56. Typically, *all* children in the class then remember *all* the facts as well as who was in charge of them.

One good way to make the assignments is to ask a child to name one fact that the child often has trouble with and then assign just that fact to that child. Periodically during the day, ask about the facts, varying how you ask.

- Sometimes, "who's in charge of 56 today?" or "who's in charge of 7 × 8" or, "who are our experts today?" followed by "what are you in charge of?"
- Freely revisit previous days' facts: "what were yesterday's facts, and who were the experts?" or "who was in charge of 49 yesterday?" and then, to that child, "what was the fact that gave us 49?" (7 × 7).

How it works

At the rate of one fact a day, the task of memorizing multiplication facts is *much* less daunting than "learn all your facts." Mastery grows rapidly. The child who is in charge *feels* "in charge," but many other children pick up that one fact at the same time, too.

Asking a child which fact he or she is in charge of—not asking "what's 7 × 8?" but literally "what fact are you today's expert on?"—puts child in charge of remembering the *whole* package, not just the "answer part." Sometimes they get 56 and must remember 7 × 8; sometimes they get 7 × 8 and must remember 56; sometimes they get nothing at all, and need to remember what they're responsible for.

See http://thinkmath.edc.org/index.php/Fact_of_the_day for "fact of the day" plan and method help children master doubles, 5×, and squares (e.g., 7 × 7). The intersection imagery helps children with multiplication by 1 and 0. "Being a specialist" makes quick work of the very few facts that remain.